

# Black hole dynamical evolution in a Lorentz-violating spacetime

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We consider the black hole dynamical evolution in the framework of a Lorentz-violating spacetime endowed with a Schwarzschild-like momentum-dependent metric. Large deviations from the Hawking-Bekenstein predictions are obtained, depending on the values of the Lorentz-violating parameter  $\lambda$  introduced. A non-trivial evolution comes out, following mainly from the existence of a non-vanishing minimum mass: for large Lorentz violations, most of the black hole evaporation takes place in the initial stage, which is then followed by a stationary stage (whose duration depends on the value of  $\lambda$ ) where the mass does not change appreciably. Furthermore, for the final stage of evolution, our model predicts a sweet slow death of the black hole, whose “slowness” again depends on  $\lambda$ , in contrast with the violent final explosion predicted by the standard theory.

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## I. INTRODUCTION

Black holes (BH’s) are very peculiar physical systems where, for the enormous forces and the extreme conditions acting there, a mere classical picture is not sufficient to describe them and we need taking into account also important quantum effects. Furthermore, the mechanical description based on general relativity must be joined with a thermodynamical treatment, because of the continuous interactions and energy exchanges with the surrounding environment. Actually, BH’s represent a proper natural laboratory for the investigation of the new edges of physics. This is particularly true for primordial and for microscopic (“mini” or “micro”) BH’s, rather than for stellar or galactic supermassive ones, because of the extreme smallness of the Planck scale at which classical and quantum approaches appear to carry quite different predictions. The most familiar and relevant quantum effect in BH physics is the so-called “evaporation”, predicted in the celebrated Hawking-Bekenstein theory (HB) [1], namely the spontaneous radiative particle emission with progressive mass loss and temperature raising, up to a final explosion without any remnant of the pre-existing BH. This evaporation is not expected in general relativity, since it is due to a quantum fluctuation on the event horizon which breaks virtual particle-antiparticle pairs, releasing the positive energy particles towards the external surroundings, and confining the negative energy antiparticles into the interior of the BH, all that resulting in an effective decreasing of the BH mass.

The observation of the final stages of the BH evaporation [2], besides to check general relativity and quantum thermodynamics in extreme and exotic physical conditions and, above all, the HB theory itself, could help to find a solution to various important open problems as, e.g.: the dark matter problem, the cosmological domain wall problem and the cosmological monopole problem [3]. Furthermore, the primordial BH density estimated from observations can severely constraint the matter and energy density fluctuations in the early universe.

Let us also remark that the final BH vanishing predicted by the HB theory is probably a non-physical extrapolation of classical results to highly non-classical regions. Actually, notwithstanding general relativity does not require a lower bound to the BH mass, it is commonly accepted that, at least in ordinary three-dimensional gravity, the minimum BH mass is of the order of the Planck mass

$$M_{\text{Planck}} = \sqrt{\hbar c^5 / G} \sim 10^{19} \text{ GeV}, \quad (1)$$

which corresponds to an event horizon with a radius of the order of the Planck length, namely about  $10^{-33}$  cm, and to a Hawking temperature of the order of  $10^{32}$  K implying a thermal energy comparable to the BH mass itself. In such conditions, the emission for Hawking evaporation of only one photon would cause the vanishing of the whole BH, from which it follows that a proper thermodynamical description could not be meaningful.

Besides the unphysical loss rate divergence, the total evaporation predicted by the standard theory entails other serious problems and inconsistencies, as the baryon and lepton number non-conservation, the “information paradox”, and the microscopical origin of the entropy [4, 5], which are essentially due to the complete evaporation of the initial BH.

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In the present paper we explore some new predictions of a physical model introduced recently by one of the Authors (G.S.) [6], where it has been shown that some of the problems mentioned above seem to be overcome by adopting a “deformed” relativity framework. There, the energy-momentum dispersion law is Lorentz-violating (LV) and the Schwarzschild-like metric is momentum-dependent with a Planckian cut-off. In such a way, net deviations of the basic thermodynamical quantities from the HB predictions have been obtained. As a matter of fact, the BH evaporation is expected to quit at a nonzero critical mass value of the order of the Planck mass, leaving a zero temperature remnant, and all the semiclassical corrections to the BH temperature, entropy, and heat capacity are divergence-free. Then, after reviewing in the following section the main results of the model proposed, in Sect. III we evaluate the modified BH evolution dynamics obtaining highly non-trivial results which, in the final section, will be analyzed and compared with the available experimental data.

## II. BLACK HOLE EVAPORATION IN A MOMENTUM-DEPENDENT SCHWARZSCHILD METRIC

In the last decades high energy (usually planckian) Lorentz violations have been proposed in many different experimental and theoretical frameworks as, e.g., (see [6] and references therein) GUT’s, causal dynamical triangulation, “extensions” of the Standard Model incorporating breaking of Lorentz and CPT symmetries, superstring theories, quantum gravity, spacetimes endowed with a non-commutative geometry. Let us point up that most of the above LV theories seem (implicitly or explicitly) to suppose an essentially non-continuous, discrete space-time where, as expected from the uncertainty relations, fundamental momentum and mass-energy scales naturally arise. Hereafter, for simplicity, we shall use the term “violation” of the Lorentz symmetry, but in some of the theories just mentioned (e.g., in the so-called “Deformed” or “Doubly” Special Relativity, where deformed 4-rotation generators are considered), although special relativity does not hold anymore, an underlying extended Lorentz invariance does exist.

In the abovementioned paper [6], following the so-called Gravity’s Rainbow (or Doubly General Relativity) approach [7, 8] which generalizes  $k$ -Minkowski Lie-algebra to noncommutative curved spacetimes, we have assumed a special LV momentum-dependent metric where, analogously to the phonon motions in crystal lattice, only at low energies any particle is allowed to neglect the quantized structure of the underlying vacuum geometry. Whilst, at very high energies, particles can effectively feel the discrete-like structure and the quantum properties of the medium crossed. Actually, a very general momentum-dependent metric can be written as

follows

$$ds^2 = f^{-2}(p)dt^2 - g^{-2}(p)d\ell^2. \quad (2)$$

The form factors  $f$  and  $g$  are expected to be different from unity only for planckian momenta, if the LV scale is assumed to be the Planck energy. One of the most important consequences of (2) is the modification of the ordinary momentum-energy dispersion law  $E^2 - p^2 = m^2$ , by means of additional terms which vanish in the low momentum limit:

$$E^2 f^2(p) - p^2 g^2(p) = m^2. \quad (3)$$

The basic consequence of the replacement of the usual metric with the “rainbow” metric in (2) is that the metric runs, i.e. we have a different metric for each energy scale. This does not contradict the principle of relativity, since a non-linear representation for the Lorentz group is adopted. As a key consequence of Eq. (3), the present model belongs to the general framework of varying speed of light theories and, as noted in [9], in such a case one should be careful in specifying how diffeomorphism invariance or lack of it works, so that the property of invariance of the varying speed of light under coordinate transformations holds true. However, diffeomorphism transformations change the metrics without changing the ratio between the speeds of photons with different frequencies. For the interested reader, an explanation of the structure of these theories, their Einstein equations, and the impact on BH solutions can be found in Ref. [10].

Taking for simplicity  $f = 1$  in Eq. (3), we can write down the above equation as follows

$$E^2 = p^2 + m^2 + p^2 F(p/M), \quad (4)$$

where  $M$  indicates a (large) mass scale characterizing the Lorentz violation. By using a series expansion for  $F$ , under the assumption that  $M$  is a very large quantity, we can consider only the lowest order nonzero term in the expansion:

$$E^2 = p^2 + m^2 + \alpha p^2 \left(\frac{p}{M}\right)^n \quad (5)$$

( $\alpha$  is a dimensionless parameter of order unity). The basic dispersion law, the most recurring in literature, is the one corresponding to the lowest exponent, i.e.  $n = 1$  (see e.g. [11, 12], and refs. therein):

$$E^2 = p^2 + m^2 + \alpha \frac{p^3}{M}. \quad (6)$$

In ref. [6] we adopted a very simple LV metric, namely

$$f^2(E) = 1, \quad g^2(p) = 1 - \lambda p, \quad (7)$$

where the positive parameter  $\lambda \sim M_{\text{Planck}}^{-1}$ . This choice is equivalent to the above first order ( $n=1$ ) LV dispersion law (6) with a negative  $\alpha$ :

$$E^2 = p^2 + m^2 - \lambda p^3. \quad (8)$$

The dispersion law (8) was also previously introduced in a paper of ours [12], in order to give a simple explanation for the baryon asymmetry in the Universe. Just because of the negative sign of the LV term, we succeeded to propose a straightforward mechanism for generating the observed matter-antimatter asymmetry through a Lorentz-breakdown energy scale of the order of the Greisen-Zatsepin-Kuzmin cut-off.

Let us stress that in current Gravity's Rainbow applications to early universe and to BH's the chosen form factors  $f$  and  $g$  do not entail a Planck cut-off and a maximum momentum. By contrast, in our metric, because of the negative term  $-\lambda p$ , the energy vanishes when  $p = p_{\max} = \frac{1}{\lambda} \sim M_{\text{Planck}}$ , which plays the role of a "maximal momentum" corresponding to the noncontinuous "granular" nature of space. Actually, because the momentum is upperly bounded by  $1/\lambda$ , the Heisenberg relations forbid to consider as physically meaningful any spatial scale smaller than  $\lambda$ , this corresponding to an effective "minimal" distance between two distinct space points, a sort of "step" of a spatial lattice that constitutes the vacuum which, at low energy, appears as really continuous. Notice that such a property directly comes from the fact that Eq. (8), differently from what happens in other models present in the literature, is *not* just the leading approximation in a series expansion in the small parameter  $\lambda$  but, rather, Eqs. (7) are assumed to be the exact form of the metric, such a form being adopted just for the related presence of a momentum cutoff. Obviously, other forms of the metric leading to this property are possible, but it is remarkable that the predictions obtained through our approach are not limited to a particular choice, but rather they seem the physical manifestation of a general result coming from the presence of a non-vanishing momentum cutoff. In particular, it is of some interest the fact that apparently model-independent features<sup>1</sup> of BH appears even in quite different approaches to quantum gravity [13].

By adopting the form factors given by (7) in the Schwarzschild metric

$$ds^2 = - \left(1 - \frac{2GM}{r}\right) dt^2 + \frac{1}{1 - \lambda p} \left[ \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \right], \quad (9)$$

in ref. [6] we obtained an important correction to the standard inverse dependence of the BH temperature with respect to the mass. Strictly speaking, in our model particles with different  $p$  move in different metrics so that, in general, a "unique" equilibrium temperature is not well-defined. However, as in ref. [6], we assume an average behavior for any particle described by a unique average

temperature for the system since, as we shall see below, no large fluctuations are expected in our model. Nevertheless, the BH mass-temperature relation shows relevant corrections with respect to the standard case. Indeed, as a matter of fact, by introducing the minimal LV mass scale ("critical mass")

$$M_{\text{cr}} \equiv \frac{\lambda \hbar c^2}{8\pi G}, \quad (10)$$

when  $M$  approaches  $M_{\text{cr}}$  the temperature sharply deviates from the HB law ( $T_{(\lambda=0)} = \hbar c^3 / 8\pi k G M$ ):

$$T = \frac{\hbar c^3}{8\pi k G M} \sqrt{1 - \frac{M_{\text{cr}}}{M}}. \quad (11)$$

Note that for masses larger than the critical value (or the Planck mass), the statistical fluctuations around the average value considered above are obviously very small, while they are non negligible for masses of the order of the critical value. However for such values, due to the Heisenberg's uncertainty relation, the photon momentum is constrained to be almost equal to the extremal cutoff value  $p_{\max}$ , since lower momenta are not allowed when the size of the evaporating BH is of the order of the Planck length. In this regime, the space structure is essentially granular, due to the presence of the cutoff  $p_{\max}$ , which forbids photon momenta greater than (about) the Planck value. Now, if the photons have, practically, the same momentum and then experience the same metric, then the BH temperature is substantially the same for any photon, as given by Eq. (11).<sup>2</sup>

Owing to evaporation, the BH temperature does not diverge at  $M = 0$  anymore, but shows a finite maximum at  $M = \frac{3}{2} M_{\text{cr}}$  in the latest moments of its life, and does vanish in the final instant of the evaporation process, at  $M = M_{\text{cr}} \neq 0$  (see Fig. 1). Actually, the most important consequence of our momentum-dependent metric is that, due to the existence of a maximum momentum, it exists a minimum mass  $M_{\text{cr}}$  for a BH, which therefore ends its life as a zero-temperature "extremal" BH with mass  $M_{\text{cr}}$ , in such a way avoiding any singularity formation and also overcoming all the problems due to the total mass evaporation.<sup>3</sup>

In the presence of Lorentz violations also the entropy shows strong deviations from the HB result  $S = 4\pi G M^2$  when the mass approaches the critical value  $M_{\text{cr}}$  (Fig. 1):

$$S \simeq \frac{4\pi k G M_{\text{cr}}^2}{\hbar c} \sqrt{\frac{M}{M_{\text{cr}}} - 1}. \quad (12)$$

<sup>2</sup> Note, instead, that a unique value of the critical mass is always present, since it is not a dynamical variable of the particles involved but, rather, a parameter of our model.

<sup>3</sup> The possibility may be considered that the cold BH remnants would be observed as WIMPs, often considered as the main components of the dark matter.

<sup>1</sup> P. Nicolini, private communication.

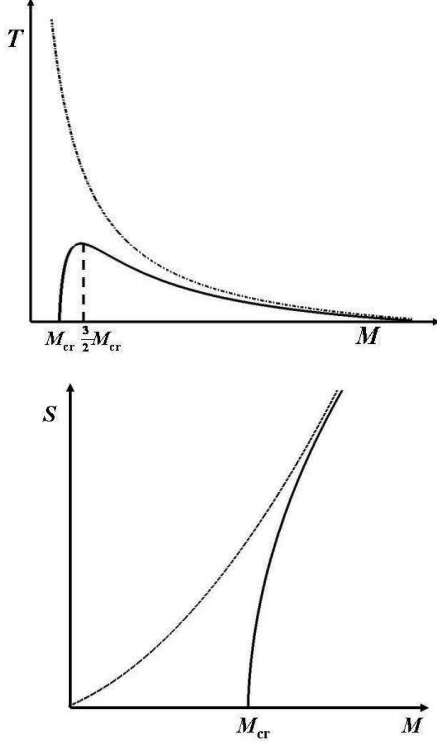


FIG. 1: BH temperature  $T$  and entropy  $S$  as a function of mass  $M$ , compared with the standard HB plot (dashed lines).

Then the entropy reaches its minimum (zero) together with the mass, for  $M = M_{\text{cr}}$ .

Also another important thermodynamical quantity, the heat capacity, diverges at  $M = \frac{3}{2}M_{\text{cr}}$  (see Fig. 2), corresponding to the maximum BH temperature, and vanishes at the minimum mass  $M_{\text{cr}}$ :

$$C \simeq \frac{16\pi k G M_{\text{cr}}^2}{\hbar c} \frac{\sqrt{\frac{M}{M_{\text{cr}}} - 1}}{3 - 2\frac{M}{M_{\text{cr}}}}. \quad (13)$$

### III. MODIFIED BLACK HOLE EVOLUTION DYNAMICS

The momentum-dependent metric adopted here obviously induces deviations from the time evolution of the BH evaporation as predicted by the Hawking theory. Here we will investigate just on such deviations, depending of course on the momentum cutoff  $1/\lambda$ .

The thermal radiance  $\mathcal{I}$  of a photon gas is given by the product of the light velocity  $v$  times the radiation energy density  $\rho$  or, in differential form,

$$d\mathcal{I} = v d\rho. \quad (14)$$

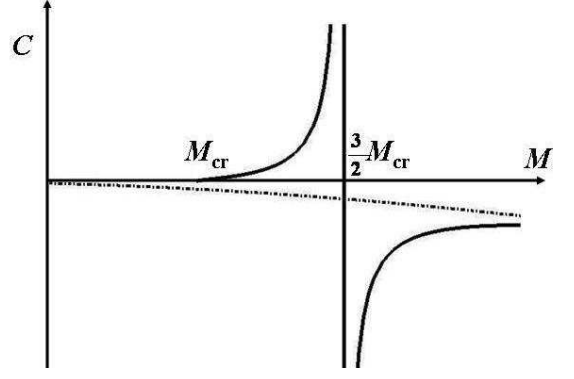


FIG. 2: Heat capacity of a BH as a function of its mass compared with the standard HB plot (dashed line).

From the dispersion relation (8) (with  $m = 0$ ), the momentum-dependent group velocity is given by

$$v = \left| \frac{dE}{dp} \right| = \frac{|2 - 3\lambda p|}{2\sqrt{1 - \lambda p}} c. \quad (15)$$

Contrary to the standard case, in our model with a momentum-dependent metric the BH radiation could be, in general, no longer thermal, so that particles in the photon gas might not be distributed in energy according to the equilibrium Bose-Einstein function. Although the correct complete procedure would be that of solving the Boltzmann transport equation for the distribution function in the present case, a simple consideration allows us to avoid such a complication within a reasonable approximation. Indeed, following the discussion after Eq. (11), it is likely to expect that the small metric variations induced by small momentum differences around the cutoff  $p_{\text{max}}$  are much less effective in destroying than fast electromagnetic interactions guaranteeing thermodynamical equilibrium. In such a case, we can certainly adopt a thermal distribution for the photons with a rescaled energy  $E = cp\sqrt{1 - \lambda p}$ , so that the energy density is given by:

$$d\rho = \frac{c}{\pi^2 \hbar^3} \frac{p^3 \sqrt{1 - \lambda p}}{e^{\frac{cp}{kT} \sqrt{1 - \lambda p}} - 1} dp. \quad (16)$$

The thermal radiance can then be written in the following general form:

$$\mathcal{I} = \frac{c^2}{2\pi^2 \hbar^3} \int_0^{1/\lambda} \frac{p^3 |2 - 3\lambda p|}{e^{\frac{cp}{kT} \sqrt{1 - \lambda p}} - 1} dp. \quad (17)$$

The lifetime equation may be obtained by considering that such radiance gives the energy power per unit area emitted by the BH,

$$\mathcal{I} = \frac{1}{A} \frac{dU}{dt}. \quad (18)$$

In terms of BH mass<sup>4</sup>  $M$  we have

$$\mathcal{I} = -\frac{c^6}{16\pi G^2 M^2} \frac{dM}{dt}, \quad (19)$$

and substituting Eq. (17):

$$\frac{dM}{dt} = -\frac{16G^2 M^2}{2\pi \hbar^3 c^4} \int_0^{1/\lambda} \frac{p^3 |2 - 3\lambda p|}{e^{\frac{c^2 p}{kT} \sqrt{1-\lambda p}} - 1} dp. \quad (20)$$

The modified mass-temperature relation to be used here is given by Eq. (11) [6]. In the standard case with  $\lambda = 0$ , Eqs. (20), (11) reduce to

$$\frac{dM}{dt} = -\frac{\hbar c^4}{3840\pi G^2} \frac{1}{M^2}. \quad (21)$$

By solving this equation for a BH with initial ( $t = 0$ ) mass  $M_0$ , the time evolution is just given by

$$M = M_0 \sqrt[3]{1 - \frac{t}{\tau_{(\lambda=0)}}}, \quad (22)$$

where

$$\tau_{(\lambda=0)} = \frac{1280\pi G}{\hbar c^4} M_0^3 \quad (23)$$

is the BH evaporation time at which  $M(t = \tau_{(\lambda=0)}) = 0$ .

For  $\lambda \neq 0$  the things sound different, since the final BH mass is no longer zero. The minimum BH mass can be simply derived by requiring the argument in the square root in Eq. (11) to be non negative, thus obtaining just the minimum mass value already reported in Eq. (10). The evaporation time  $\tau$  is then now defined by  $M(t = \tau) = M_{\text{cr}}$ , and can be obtained by the solution of Eqs. (20), (11). To this end, it is useful to replace the variables  $T, M$  by the dimensionless quantities

$$\xi = \frac{c}{\lambda k T}, \quad w = \frac{M}{M_{\text{cr}}}, \quad (24)$$

in terms of which Eq. (20) becomes

$$\frac{dw}{dt} = -\frac{w^2 \gamma(\xi(w))}{\tau_0}. \quad (25)$$

Here  $\tau_0 = \pi^2 \hbar^2 c^2 \lambda^3 / 2G$  is a typical reference time value, while

$$\gamma(\xi) = \frac{1}{2} \int_0^1 \frac{y^3 |2 - 3y|}{e^{\xi y \sqrt{1-y}} - 1} dy \quad (26)$$

(note that  $\xi(w) = w\sqrt{w}/\sqrt{w-1}$ ). The BH time evolution may be obtained by Eq. (25) or, more conveniently,

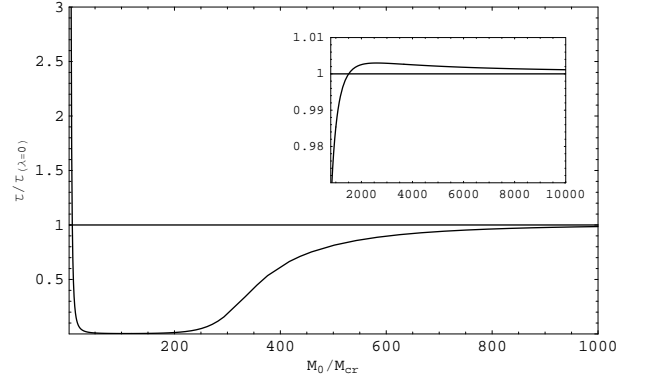


FIG. 3: BH lifetime versus initial mass normalized to the critical value  $M_{\text{cr}}$ . In the insert we show the peculiar behavior for very large  $M_0/M_{\text{cr}}$ : the standard value is approached asymptotically from the above, and not from below.

by evaluating numerically the integral in the equivalent equation

$$\frac{t}{\tau_0} = \int_w^{w_0} \frac{dw}{w^2 \gamma(\xi(w))}. \quad (27)$$

The evaporation time  $\tau$  for which the BH mass reduces to  $M = M_{\text{cr}}$  is then given by

$$\tau = \frac{\pi^4}{5} \frac{1}{w_0^3} \tau_{(\lambda=0)} \int_1^{w_0} \frac{dw}{w^2 \gamma(\xi(w))}. \quad (28)$$

For small  $\lambda$ , Eq. (28) is approximated by the following expression,

$$\tau = \tau_{(\lambda=0)} \left[ 1 + 3 \left( 1 + \frac{270\zeta(5)}{\pi^4} \right) \frac{M_{\text{cr}}}{M_0} \right], \quad (29)$$

revealing that in such a case the BH lifetime is *larger* than that predicted in the standard theory, as naively expected from the fact that the final BH mass is now non-vanishing. In general, however, the situation is more interesting than that, as it is evident from Fig. 3. Indeed, we find that for initial masses  $M_0 \lesssim 10 M_{\text{cr}}$ , the evaporation time may be much greater than the standard value, while for  $50 M_{\text{cr}} \lesssim M_0 \lesssim 200 M_{\text{cr}}$  we have  $\tau \ll \tau_{(\lambda=0)}$ , reaching a minimum of about  $\tau/\tau_{(\lambda=0)} \sim 0.001$ . Instead, the standard value of the BH lifetime is approximately recovered for  $M_0 \gtrsim 400 M_{\text{cr}}$ .

A non-trivial BH evolution dynamics is also evident from the graphs in Figs. 4 where the BH lifetime is plotted as a function of the initial mass (in Planck units) for different values of the parameter  $\lambda$ . The sensitivity of the BH dynamics to this parameter is, instead, deduced from Fig. 5. Note that the apparent cutoffs for large  $\lambda$  (for the considered values of  $M_0$  are just a manifestation of the existence of a minimum BH mass, whose value depends on  $\lambda$  through Eq. (10). It is then evident that, for large  $\lambda$  (compared to the Planck scale), BH's of mass lower than

<sup>4</sup> We have, obviously,  $U = Mc^2$ ; the BH surface area  $A$  equals  $4\pi$  times the Schwarzschild radius squared.



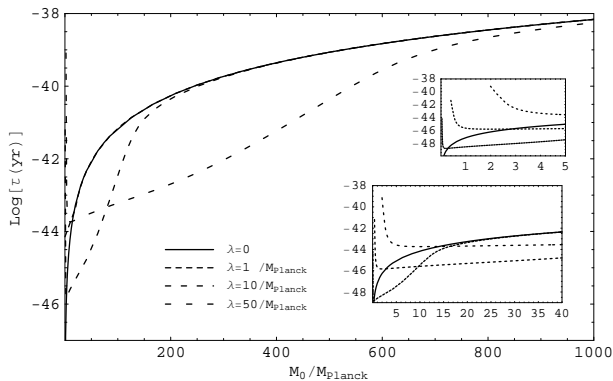


FIG. 4: BH lifetime (in absolute units) as function of the initial mass  $M_0$  (in Planck units) for different values of the Lorentz violating parameter  $\lambda$ . The endpoints of each curve for small  $M_0$  (see the inserts) correspond to the critical values  $M_{cr}$  for the considered  $\lambda$ .

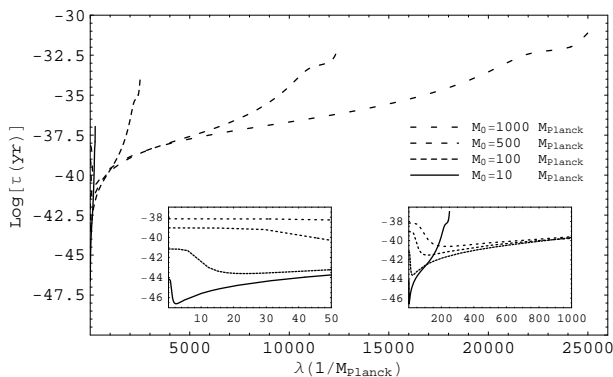


FIG. 5: BH lifetime (in absolute units) as function of the Lorentz violating parameter  $\lambda$  (for small values, see the inserts) for different values of the initial mass  $M_0$  (in Planck units). The endpoints of each curve correspond to the non-vanishing final value  $M_{cr}$  for the BH mass.

the value in Eq. (10) *cannot be formed* and, consequently, no evaporation takes place at all.

The entire dynamics of the BH evaporation may be followed by plotting BH mass versus time; this is done here in Fig. 6 for a typical initial mass value of  $M_0 = 100 M_{\text{Planck}}$ . Obviously, different results are obtained for different values of the Lorentz-violating parameter  $\lambda$  but, as shown in the mentioned figure, the dependence on  $\lambda$  is not monotonic. For example, for the given value of  $M_0$ , the mass decrement before the final stage is slower than in the standard case for values of  $\lambda$  up to approximately  $20/M_{\text{Planck}}$  (the curves in Fig. 6 labeled with  $\lambda = 3, 5, 10, 15/M_{\text{Planck}}$  lie above the one corresponding to the standard case with  $\lambda = 0$  for most of the BH life). The reverse is true for  $\lambda$  greater than  $20/M_{\text{Planck}}$  and, in particular, for large values of  $\lambda$ , most of the BH evaporation takes place in the initial stage, then followed by a

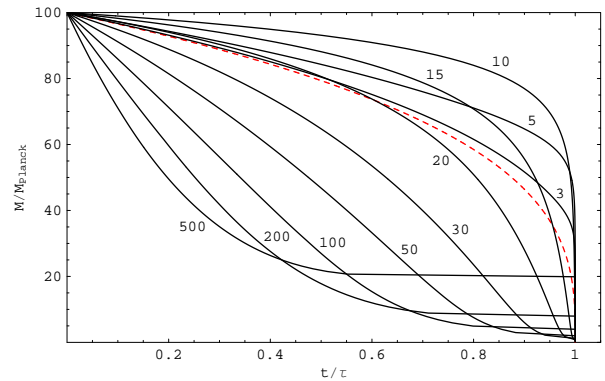


FIG. 6: BH mass (in Planck units) versus time (normalized to BH lifetime) for an initial mass value of  $M_0 = 100 M_{\text{Planck}}$ . The numbers close to each curve gives the value of  $\lambda$  in  $M_{\text{Planck}}^{-1}$  units. The dashed curve correspond to the standard case with  $\lambda = 0$ .

stationary stage (lasting even for more than half of the BH life) where the mass does not change appreciably.

Furthermore, a very characteristic feature of the BH evolution arises in our model, regarding the final stage of the life of a BH. In the standard theory, the BH evolution ends with a final explosion, characterized by an infinite value of the derivative  $dM/dt$ ; that is, the curve  $M(t)$  for  $\lambda = 0$  has a vertical tangent for  $t = \tau$ . This never occurs for  $\lambda \neq 0$ . In order to show this, we have enlarged the plot in Fig. 6 around  $t = \tau$ , and the result is shown in Fig. 7. It is evident that, even for small values of  $\lambda$ , the slope of the  $M(t)$  curve is always smaller than that in the standard case when approaching the limiting value  $t = \tau$ . For very small  $\lambda$ , the stationary stage discussed above is increasingly shorter, then recovering the standard result for  $\lambda \rightarrow 0$ . Thus our model predicts a BH sweet slow death without any final explosion, the “slowness” of this stage depending on the value of the Lorentz violating parameter  $\lambda$ .

#### IV. CONCLUSIONS

Starting from our previous introductory work on the subject, we have studied the time evolution of the BH evaporation in a Lorentz-violating spacetime endowed with a Schwarzschild-type momentum-dependent metric. We have computed the BH mean lifetime obtaining net deviations from the HB predictions. Actually, the BH evaporation dynamics results to be ruled by a non-vanishing minimum mass  $M_{cr}$  which is a function of the Lorentz-violating parameter  $\lambda$ . We have indeed found that, independently of the strength of Lorentz, for an initial mass of the order of the Planck mass, the BH lifetime can result very larger than the HB prediction  $\tau_{(\lambda=0)}$ ; while, for  $M_0$  of the order of  $100 M_{cr}$ , the lifetime results  $10^3$  times

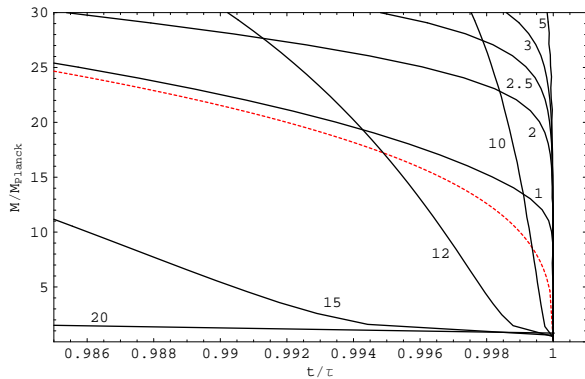


FIG. 7: The same as in Fig. 6 but enlarged near the final point  $t = \tau$ . Even for small values of  $\lambda$ , the slopes of the curves are smaller than in the standard case (dashed curve); here this is particularly evident for  $\lambda \geq 10$ .

smaller than  $\tau_{(\lambda=0)}$ . The standard prediction is recovered only for  $M_0 \gtrsim 400 M_{\text{cr}}$ .

Furthermore, we have shown that for large Lorentz violations most of the evaporation takes place at the beginning; then a quasi-stationary era follows where the mass decreases very slowly.

Actually, an important result achieved in the present work regards the final instants of the BH life. We have indeed found that toward the lifetime end, it is always present an evaporation damping, which starts earlier or later depending on the magnitude of  $\lambda$ . Consequently, by contrast with Hawking's predictions, our theory does entail a slow death of terminal black holes instead of an infinitely fast evaporation, resulting in a dramatic final gamma-ray burst. This might affect the experimental observation of the Hawking radiation coming out from very light BH's as the so-called "mini-BH's".

Mini-BH's are supposed to have been created by primordial density fluctuations in the early universe and, according to the HB theory, undergo a sufficiently intense evaporation to be detected by means of suitable experimental devices. Actually, because of evaporation, primordial BH's with initial mass of the order of  $10^{15}$  g (the mass of a typical asteroid) and with initial Hawking temperature of about  $10^{11}$  K would explosively vanish just today (while lighter BH's would be already totally evaporated). At variance, BH's endowed with a mass larger than  $M_{\odot}$ , as the ones produced during the gravitational collapse of a star, according to HB theory have a lifetime enormously larger than  $10^{10}$  years, i.e. larger than the estimated age of our Universe. On the other hand, supermassive BH's, as galactic ones or BH's emerging from star clusters collapse, endowed with a mass equal or larger than  $10^6 M_{\odot}$ , have a temperature still smaller with respect to the actual cosmic microwave background radiation, so that they are not allowed to evaporate because of the second law of thermodynamics.

Notably, theories which involve additional space di-

mensions where (only) the gravitational force can act—as, e.g., string theory or braneworld gravity theories with extra-dimensions [14–16, 18–21]—do carry significant deviations from the usual BH evaporation dynamics. In fact, primordial BH's are more easily produced in the Randall-Sundrum theoretical framework and, in the presence of extra spatial dimensions, should undergo a more explosive phase of evaporation. The increasing of the available number of particle modes for the evaporation or some exotic planckian events (as, e.g., phase transitions from black string to BH) should result in a strong raise of the Hawking radiation luminosity. In some primordial universes endowed with almost an extra dimension the temperature of primordial BH's turns out to be lowered, and the evaporation appears slowing down with longer lifetime, so that lighter primordial BH's, not yet evaporated, could be detected even in our galaxy. Furthermore, if LHC is not far above the re-scaled Planck energy (which results lowered to TeV scale in the just mentioned theories), micro-BH's could be abundantly produced in the ATLAS experiment: at the expected luminosities, BH's production rate can be estimated to be of the order of 10 pb, and we could observe several BH's per minute or per second. With a mass of few TeV, the produced BH's would be extremely hot and evaporate almost instantaneously, emitting Hawking radiation composed by particle-antiparticle pairs for all allowed degrees of freedom, at equal rates.

Instead, according to HB theory, primordial BH's with mass about  $10^{15}$  g might be detected by observing high energy gamma rays produced in the last instants of their lives, which should end with a violent explosion equivalent to many teraton hydrogen bombs (typical expected values of the order of  $10^{34}$  erg per  $\mu\text{s}$ ).<sup>5</sup> Also with the aim of detecting gamma ray bursts emitted by dying BH's, NASA has recently (2008) put in orbit around the Earth the *Fermi Gamma-ray Space Telescope* (FGST). Actually, in our approach we just predict a progressively vanishing evaporation for a terminal mini-BH ending in a cold burnt-out remnant. Therefore, contrary to what abovesaid, because of a very weak terminal Hawking radiation, the actual detection techniques, also by making recourse to powerful space telescopes, might not be able to observe and measure the weak evaporation of primordial BH's dying nowadays. For analogous reasons, depending on the initial mass and on  $\lambda$ , the evaporation rate versus time for micro-BH's produced in ultra-high energy particle physics laboratories might involve very weaker signals and very fewer decay products with respect to what currently expected. On the other hand, at the present no such micro-BH event has been observed at CERN. In the last fifteen years various high-energy

<sup>5</sup> Furthermore, it has been suggested that a small BH (of sufficient mass) passing through the Earth would produce a detectable acoustic or seismic signal [17].

gamma-ray observations have been performed. as e.g. SAS-II [22], COS-B [23], EGRET [24, 25]. As a matter of fact, in those investigations as well as in the last data from LAT e GBM on FGST, no definitive experimental evidence of terminal BH evaporation via Hawking radiation has been found [25–27]. Apart some recent re-interpretations of very short gamma-ray bursts in outdated experiments as KONUS [28], the current studies present in literature do not strictly relate high energy

astrophysical events to strong Hawking radiation pulses, but only upperly constraint the apparent density of terminal mini-BH's. This turns out to be quite consistent with our present predictions on BH time evolution.

Several other interesting cosmological implications deserve the application of our present model; these, however, will be the subject of future studies.

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